AD 297 444

63-2-5

SP-116

John E. Walsh

A General Simulation Model for Logistics Operation in a Randomly-Damaged System

System Development Corporation/2500 Colorado Avenue/Santa Monica, California

SP-116 John E. Walsh

A General Simulation Model for Logistics Operation in a Randomly-Damaged System

January 10, 1960

SYSTEM DEVELOPMENT CORPORATION, SANTA MONICA, CALIFORNIA

SUMMARY

The logistics operation considered is imbedded in a system that is subject to damage. Natural attrition and induced attrition can both occur. This system is concerned with the availability and/or use of several different types of items during a specified period of time. The purpose of logistics is to furnish transportation, reduce natural attrition through maintenance, and return damaged items to operational condition. The capability of the logistics operation is examined by simulating the operation of the entire system for the specified time period. The damage received by the various parts of the system depends on the situation at the start of the time period, the use made of the items, the nature of the induced attrition, and maintenance procedures, etc. Attrition occurs on a probability distribution basis and is introduced by a Monte Carlo procedure. The time required to return a damaged item to operational condition depends on the type of damage, the damage received by other items, the characteristics of the logistics operation, damage received by the logistics part of the system, etc. To obtain the simulation model, the overall system is divided into subsystems on the basis of the use and location of the items, the properties of the logistics operation, etc. The time scale used is discrete; i.e., all important effects are considered to occur during one of a specified finite set of time intervals. The same form of simulation model is used for representing the operation of each subsystem for each time interval. That is, the mathematical model has the same functional form but the variables in this functional form can differ with the subsystem and time interval. These subsystems can interact in many ways; e.g., items can be transferred between subsystems, damage received in one subsystem can affect the logistics operations in other subsystems, etc. Using time as the basis for coordination among subsystems, the mathematical model and random assignments for attrition can be programmed on a high speed computer. The simulation is performed by first obtaining the operational results for all subsystems at the first time considered; then, using the results for the first time, the results are obtained for the second time; etc. Examination of the subsystem results for each time interval should furnish a good indication of the capabilities of the logistics operation. Repetitions of this simulation furnish an indication of the effects of statistical variation.

INTRODUCTION

In practice, logistics operations are virtually always parts of systems which are devoted to the accomplishment of one or more specified purposes. Then the suitability of a given logistics operation, or the relative desirabilities of several competitive logistics operations, ordinarily can not be determined independently of the system characteristics. In fact, the worth of a logistics operation seems to be most meaningfully evaluated by consideration of the behavior of the system when this logistics operation is used.

When a system is extensive and complicated, a deterministic examination of its detailed operation is seldom feasible, even by the use of simulation and high-speed computers. If allowance is also made for random effects, such as damage, the difficulty of examining the system operation in detail is greatly increased. However, by suitable aggregation of detail, so that only the important effects are considered, the principal characteristics of the operation of a system can often be simulated in a feasible manner, even when random damage occurs. The purpose of this paper is to present a model of a somewhat general nature for performing simulations of this type.

For the simulation model presented, the system in which the logistics operation is imbedded is not of a completely general nature. The viewpoint adopted is that the purpose of the simulation is to examine the logistics operation under realistic circumstances. This purpose can often be accomplished to a reasonable extent by limiting consideration to systems which are concerned with the use and availability of various types of items and for which the "strategies" for system operation are restricted to transfer of items within the system and to maintenance and repair capabilities. Here natural and induced damage to the items are the only random effects that are introduced, and these depend on probability distributions which, during the time period considered, are influenced only by happenings within the system.

The major loss of generality in the model presented is that, during the time period considered, the probability distributions for induced attrition are influenced only by occurrences within the system containing the logistics operation. Often the induced attrition arises from a separate system. Then the interactions between the two systems during the time interval, and their effect on the operation strategies adopted by the systems, can be important in evaluating the behavior of the system containing the logistics operation. However, when the principal interest is in the logistics operation, the restricted system considered in this paper should be satisfactory in most cases. Actually, by judicious specification of the functional forms for the probability distributions of induced attrition, the effects of the second system can usually be at least roughly approximated. When a realistic evaluation of the behavior of the entire system is desired, a separate simulation model can be developed for the system influencing the induced attrition. Then the combination of the two systems can be simulated by relating the two models through their strategies and the probability distributions for induced damage.

In the model, damage can occur as the result of induced attrition and/or natural attrition. As opposed to induced attrition, which arises from unusual causes and can be deliberately introduced by another system, natural attrition represents the random damage that occurs in the ordinary day-to-day operation of the system.

A characteristic of the simulation model presented is that some of the strategies (those involving transfer of items) can be chosen on the basis of an estimation of the future behavior of the system. That is, suppose that the system operation has been simulated up to the end of a given time interval; using these observed results, the future system behavior is anticipated by assuming that the attrition levels that have been observed are the actual attrition levels for future time intervals. This procedure generalizes the less realistic procedure of fixing these strategies prior to the simulation and includes the fixed strategy situation as a special case.

A fundamental part of the development of the simulation model is the conversion of the continuous situation for system operation to an approximately equivalent discrete situation which is feasible for simulation on a high-speed computer. This is accomplished by dividing the time period during which simulation is to be performed into time intervals of equal length and by dividing the system into appropriate subsystems. The selection of the subsystems and the basic time interval depends on the system considered and is determined on a judgment basis.

Every subsystem is concerned with the operation and the transportation of items. Some of the subsystems are also concerned with the performance of maintenance and/or the repair of damage for items. All operations of the system occur as a result of the use of the items in the system. These operations include the logistics functions of transportation, maintenance, and repair; in fact, some of the items are introduced exclusively for the logistics part of the system. Also some of the subsystems are included for special purposes; that is, some subsystems are introduced for the performance of maintenance and/or repair while others are included for the removal from the system or the introduction into the system of items.

Starting with the initial condition of the system (specified), the simulation is performed over all subsystems for the first time interval. Here, and at all steps, the damage actually received is introduced by a Monte Carlo procedure. On the basis of the initial conditions and the results at the end of the first time interval, the simulation is conducted for the second time interval. In a stepwise fashion, this procedure is continued until the condition of the system at the end of the final time interval is determined. The totality of results, for all the time intervals, constitutes a simulation of the behavior of the system. Several independent repetitions of this simulation should furnish a good indication of the statistical variation in system operation that is due to the random character of the damage received.

In the mathematical model, each item that occurs in the system operation is considered separately. Roughly stated, the breakdown used for the model is made with respect to the time interval considered, the item considered, the type of item, the subsystem location of the item, the status (e.g., in operation, in transit to repair, in maintenance) of the item, the maintenance schedule for the item, and the anticipated future-location scheduling for the item. Of these, the time interval considered and the location for the item considered receive the principal attention in the model. In fact, as indicated by the relationships that are used to state the mathematical model, the same form of simulation model can be used for each combination of time interval, item, and item location; that is, with little loss in generality, the mathematical model can be considered to have a fixed functional form which involves variables that depend on the time interval, item, and item location. Having fixed functional forms of this nature is convenient from the viewpoint of simulation on a high-speed computer. Namely, the problem is reduced to that of determining the values of the variables that occur in the fixed functional forms.

The specification of the mathematical model is made in general terms. That is, the available values that are sufficient to determine a new value are always explicitly identified; however, the function of these values that furnishes the new value is seldom explicitly stated. The model is sufficiently general to represent many situations of interest but does not concern itself with the problem of explicitly determining the various functions that occur in its statement. For each given system to be simulated, this problem would ordinarily be solved by a technical examination of the properties and the purposes of the system.

The model presented represents an appreciable aggregation of detail. However, in spite of this consolidation, a very large number of evaluations must be made for each time interval. In fact, unless care is exercised in selecting the basic time interval, the subsystems, the types of items, the functional forms, the levels of maintenance considered, the levels of damage that can be received, etc., even the model presented may be too massive for simulation on any presently available high-speed computer.

Simulations of the type considered seem to be most valuable for developing an understanding of a logistics operation that is imbedded in a complicated system. These simulations should also be useful in examining the relative desirability of stated logistics strategies and the relative desirability of specified logistics operations.

The next section, REMARKS CONCERNING THE MODEL, contains statements of a more detailed nature about the characteristics of the simulation model. The quantities that occur in the model are identified and defined in the following section, which is titled DEFINITION OF NOTATION. The next to last section, ASSUMPTIONS AND CONDITIONS, contains statements concerning limitations of the model and about some of its properties. The final section, FUNCTIONAL RELATIONSHIPS AND MATHEMATICAL MODEL, contains the statement of the general simulation model.

REMARKS CONCERNING THE MODEL

The mathematical model presented for simulation uses involves so many different quantities and is of such a complicated nature that no attempt will be made to give a detailed word description of all its properties. The only description given that covers all the details of the model is of a technical character; the next three sections of the paper are devoted to this description. However, discussions do seem to be warranted for some of the properties of the model and these discussions are given in this section.

In addition to the transfer schedules, various other quantities in the model have the characteristics of "strategies"; that is, these quantities have influence on the system operation and, at least to some extent, can be deliberately specified. Some of these strategies are included in the condition of the system at the beginning of the time period. Some of the others are represented by the explicit statements of the functions that are used in parts of the mathematical model; that is, these functional forms are determined by the characteristics of the system and improvement of these characteristics is one method of improving the system operation. For example, the functional forms for the times spent in transit, in maintenance, and in repair represent strategies of this type. The maintenance schedules specified for the items represent stragies of another type, since they influence the probability distributions for natural attrition. Investigation of the effects of the strategies on the system behavior is perhaps the most important reason for simulating the system operation.

The maintenance schedule for an item is expressed in terms of the accumulated amount of operational time for that item (accumulated from the beginning of the time period being simulated). However, an interval of time during which an item is in operational condition does not necessarily represent a full interval of operational time for that item. In fact, the amount of operational time accumulated during a time interval in which an item is in operational condition can depend on the time interval considered, the subsystem location of the item, the item type, whether the item is in transit, the number of other items of this type in this subsystem at this time, etc. An item accumulates no operational time during a time interval if it is not in operational condition at both the beginning and the end of this interval. Although accumulated operational time is not necessarily expressed as an integral number of time intervals, the maintenance schedules are expressed in this fashion. An item which was in an operational status at the beginning of a time interval needs maintenance at the end of this interval if its accumulated operational time equals or exceeds a critical value in the maintenance schedule at the end of the time interval.

At the end of each time interval, the transfer schedules that are to be introduced for items are chosen by anticipating the future behavior of the system. Since the system operation for future time intervals also involves the selection of transfer schedules at these future times, the amount of computation required in estimating the future behavior can easily reach an infeasible level. In many cases, adopting specified schedules for the items that are not used in the logistics operation may be desirable. Restricting the opportunity to make schedule changes to every third or fourth time interval represents another possibility for reducing the amount of computation. An important part of the mathematical model is an efficiency criterion for simultaneously deciding, at the end of each time interval, which combination of the eligible transfer schedules is to be used. A satisfactory specification of this criterion, for each time interval, represents a difficult problem that must be solved for each system that is simulated.

For items of a given type, the damage that can be received by an item is represented by a finite set of levels; also a common set of levels is used for representing the kinds of maintenance that an item can be scheduled to undergo. To simplify the presentation, the number of damage levels is considered to be the same for all types of items and the number of maintenance levels is taken to be the same for all types of items. This simplification results in no loss of generality since not all of the levels need to be used for every type of item. In general, the meanings of the levels may differ greatly from one type of item to another. The procedure of using levels is evidently suitable for maintenance and should nearly always furnish an acceptable representation of damage, if enough damage levels are used.

At the end of any given time interval, the level of future natural damage and the level of future induced damage can each be estimated as the average of the observed values over all the time intervals up to and including the given interval. Here the estimates are for the average damage during a single time interval and are obtained for every combination of item type and subsystem. Then, the anticipated future behavior of the system is predicted by starting with the situation at the end of the given time interval and simulating the future system operation under the assumption that the observed average values equal the actual damage values for each of the future time intervals. Here probability enters only in a very restricted sense, and is introduced by a Monte Carlo process. Specifically, let an average attrition value be stated in the form I + f, where I is a non-negative integer and f is a non-negative proper fraction; then the damage level actually used is selected on a Monte Carlo basis and has the value I with probability 1 - f and the value I + 1 with probability f. This random rounding of an average damage value to one of the damage levels considered can be done (independently) for each combination of item and future time interval. In this manner, a definite damage level is determined for each item at each future time interval.

For a given subsystem, a different type of logistics item might be needed for each combination of logistics task and item type. Sometimes, however, one type of logistics item performs the same logistics function for items of several types. Consequently, groupings of items of several different types can occur in the determination of the effects of numbers of items on the logistics operation for a subsystem.

Finally, the wide range of application for the simulation model presented should probably be emphasized. The principal restrictions imposed by the mathematical model are of a general nature; namely, statements of which available values are sufficient for the determination of a new value. Thus the model is applicable for the simulation of any system for which a basic time interval, a set of subsystems, etc., can be determined so that the relations actually satisfied (to a reasonable approximation) depend on the same sets of variables as those for the corresponding relations in the model. Of course, if only a subset of the set of available values specified by the model is needed for determining the new value, the relation stated in the mathematical model is satisfied.

DEFINITION OF NOTATION

Extensive use is made of symbolism in presenting the functional form of the mathematical model for a specified subsystem at a given time. Following are the definitions for this notation. These definitions should also be useful in identifying the factors that are explicitly considered in the simulation model presented.

t = time interval considered ($t = 1,\ldots,T$)

u = identification number for an item ($u = 1,\ldots,U$)

v_u = classification type for u-th item ($v_u = 1,\ldots,V$). Here types $1,\ldots,L$ refer to non-logistics items while types $L + 1,\ldots,V$ refer to logistics items

$j_u(t)$ = subsystem location of u-th item during the t-th time interval. The subsystems are identified by the numbers $1,\ldots,J$; $j_u(0)$ is specified initial condition

$s_u(t)$ = status of u-th item at end of t-th time interval; $s_u(0)$ is specified initial condition. Seven different status levels are considered:

Level 1 $\equiv$ item in operation (not in transit)

Level 2 $\equiv$ item operational but in transit

Level 3 $\equiv$ item in transit to receive maintenance

Level 4 $\equiv$ item in transit for repair of damage that is at different level from that present at end of previous time interval

Level 5 $\equiv$ item in transit for repair of damage that is at same level as that present at end of previous time interval

Level 6 $\equiv$ receiving maintenance

Level 7 $\equiv$ receiving repair

$T_o^{(u)}(t)$ = accumulated amount of operational time at end of t-th interval; $T_o^{(u)}(0) = 0$

$m_u^{(s)}$ = maintenance schedule for u-th item; depends on accumulated amount of operational time for that item

$M_u[T_o^{(u)}(t)]$ = maintenance level needed by u-th item at end of t-th time interval. Maintenance levels considered are 0,1,...,M where level 0 denotes that no maintenance needed

$d_N^{(u)}(t)$ = level of damage from natural attrition received by u-th item during t-th time interval (in addition to induced damage received in this time interval and damage present at beginning of interval). Levels considered for damage from any source are 0,1,...,D, where level 0 denotes that no damage received

$d_I^{(u)}(t)$ = level of damage from induced attrition received by u-th item during t-th time interval (in addition to natural damage received in this time interval and damage present at beginning of interval)

$P_N^{(u)}(d;t)$ = probability that u-th item will receive natural damage of level d during t-th time interval ($d = 0,1,\ldots,D$). The value of $d_N^{(u)}(t)$ is a random choice from this discrete probability distribution

$P_I^{(u)}(d;t)$ = probability that u-th item will receive induced damage of level d during t-th time interval $(d = 0,1,\ldots,D)$. The value of $d_I^{(u)}(t)$ is a random choice from this discrete probability distribution

$D_N^{(u)}(t,j)$ = arithmetical average of $d_N^{(u')}(t')$ over all $t' \le t$ and all u' such that $v_{u'} = v_u$ and $j_{u'}(t') = j$

$D_I^{(u)}(t,j)$ = arithmetical average of $d_I^{(u')}(t')$ over all $t' \le t$ and all u' such that $v_{u'} = v_u$ and $j_{u'}(t') = j$

$d_u(t)$ = overall damage level of u-th item at end of t-th time interval; $d_u(0)$ is specified initial condition

$t_o^{(u)}(t)$ = anticipated number of additional time intervals, after t-th, required for u-th item to become operational; here $t_o^{(u)}(t) = 0$ implies that $s_u(t) = 1$ or 2. Determined by starting with situation at end of t-th interval and simulating results for future time intervals under assumption that $d_N^{(u')}(t') = D_N^{(u')}[t,j_{u'}(t')]$ and $d_I^{(u')}(t') = D_I^{(u')}[t,j_{u'}(t')]$ for $t' > t$ and all u'; set equal to $T + 1 - t$ if item not anticipated to be in operation by end of T-th interval

$t_C^{(u)}(t)$ = anticipated number of additional time intervals, after t-th, required for u-th item to complete transit (change of subsystems); here $t_C^{(u)}(t) = 0$ implies that u-th item not in transit at end of t-th interval. Determined by starting with situation at end of t-th interval and simulating results for future time intervals under the assumption that $d_N^{(u')}(t') = D_N^{(u')}[t,j_{u'}(t')]$ and

$d_I^{(u')}(t') = D_I^{(u')}[t, j_{u'}(t')]$ for $t' > t$ and all u'; set equal to $T + 1 - t$ if transit not anticipated to be completed by end of T-th interval

$t_m^{(u)}(t)$ = anticipated number of additional time intervals, after t-th, required for maintenance to be completed for u-th item; here $t_m^{(u)}(t) = 0$ implies that u-th item not receiving maintenance at end of t-th interval. Determined by starting with situation at end of t-th interval and simulating results for future time intervals under the assumption that $d_N^{(u')}(t') = D_N^{(u')}[t, j_{u'}(t')]$ and $d_I^{(u')}(t') = D_I^{(u')}[t, j_{u'}(t')]$ for $t' > t$ and all u'; set equal to $T + 1 - t$ if maintenance not anticipated to be completed by end of T-th interval

$t_r^{(u)}(t)$ = anticipated number of additional time intervals, after t-th, required for repair to be completed for u-th item; here $t_r^{(u)}(t) = 0$ implies that u-th item not receiving repair at end of t-th interval. Determined by starting with situation at end of t-th interval and simulating results for future time intervals under the assumption that $d_N^{(u')}(t') = D_N^{(u')}[t, j_{u'}(t')]$ and $d_I^{(u')}(t') = D_I^{(u')}[t, j_{u'}(t')]$ for $t > t'$ and all u'; set equal to $T + 1 - t$ if repair not anticipated to be completed by end of T-th interval

$j_m^{(u)}(t)$ = subsystem in which u-th item receiving maintenance during t-th time interval; here $j_m^{(u)}(t) = 0$ implies that u-th item not receiving maintenance during t-th interval

$j_r^{(u)}(t)$ = subsystem in which u-th item receiving repair during t-th time interval; here $j_r^{(u)}(t) = 0$ implies that u-th item not receiving repair during t-th interval

$n_1(t,v,j)$ = number of operational items of type v (status 1 or 2) in subsystem j at end of t-th time interval which were in status 1 at beginning of this interval

$B_v(t,j)$ = lower bound for number of items of type v that are desired to be in operation (status 1) in subsystem j at end of t-th time interval

$n_s(t,v,j_1,j_2)$ = number of items of type v and in operation (status 1) in subsystem j_1 at end of t-th time interval that are scheduled to begin transit to subsystem j_2 at end of t-th interval; schedule can be stated in terms of $n_1(t,v,j)$. Several possible schedules can be specified for $n_s(t,v,j_1,j_2)$ with the schedule selected being determined by the anticipated future behavior of the system for each possibility, under the assumption that $d_N^{(u)}(t') = D_N^{(u)}[t,j_u(t')]$ and $d_I^{(u)}(t') = D_I^{(u)}[t,j_u(t')]$ for $t' > t$ and all u. These possible schedules are denoted by $n_s^{(k_s)}(t,v,j_1,j_2)$, $k_s = 1,\ldots,K_s(t,v,j_1,j_2)$

$n_a(t,v,j_1,j_2)$ = number of items of type v and in operation (status 1) in subsystem j_1 at end of t-th time interval that actually begin transit to subsystem j_2 at end of t-th interval

$q_u[t,v_u,j_u(t),j_2]$ = queuing priority assigned u-th item in selection of which items of type v_u and status 1 at end of t-th time interval are to start transit from subsystem $j_u(t)$ to subsystem j_2 at end of t-th interval; $q_u[t,v_u,j_u(t),j_2]$ is positive for $s_u(t) = 1$ and zero otherwise

$S_o(t,v,j)$ = specification of the in-transit subsystem locations, in given order, for an operational item (status 2) of type v that began transit from subsystem j and at end of t-th time interval; last of these subsystems is destination of item. The item changes from status 2 to status 1 when the destination subsystem, denoted by $J_o(t,v,j)$, reached in operational condition. Several alternative specifications of in-transit subsystem locations could be given with specification selected being determined by the anticipated future behavior of the system under the assumption that $d_N^{(u)}(t') = D_N^{(u)}[t,j_u(t')]$ $d_I^{(u)}(t') = D_I^{(u)}[t,j_u(t')]$ for $t > t'$ and all u. These alternatives are denoted by $S_o^{(k_o)}(t,v,j)$, $k_o = 1,\ldots,K_o(t,v,j)$

$S_M(t,v,m,j)$ = specification of the in-transit subsystem locations, in given order, for an item of type v needing maintenance at level m which began transit from subsystem j and at end of t-th time interval; last of these subsystems is destination of item for receiving maintenance. The item changes from status 3 to status 6 when the destination subsystem reached in undamaged condition. Several alternative specifications of in-transit subsystem locations could be given with specification selected being determined by the anticipated future behavior of the system under the assumption that $d_N^{(u)}(t') = D_N^{(u)}[t, j_u(t')]$ and $d_I^{(u)}(t') = D_I^{(u)}[t, j_u(t')]$ for $t > t'$ and all u. These alternatives are denoted by $S_M^{(k_M)}(t,v,m,j)$, $k_M = 1, \ldots, K_M(t,v,m,j)$

$S_R(t,v,d,j)$ = specification of the in-transit subsystem locations, in given order, for an item of type v needing repair of level d damage which began transit for repair of this level of damage from subsystem j and at end of t-th time interval. Several alternative specifications of in-transit subsystem locations could be given with specification selected being determined by the future behavior of the system under the assumption that $d_N^{(u)}(t') = D_N^{(u)}[t, j_u(t')]$ and $d_I^{(u)}(t') = D_I^{(u)}[t, j_u(t')]$ for $t > t'$ and all u. These alternatives are denoted by $S_R^{(k_R)}(t,v,d,j)$, $k_R = 1, \ldots, K_R(t,v,d,j)$

$e(t)$ = efficiency criterion for simultaneously selecting the strategies $n_s(t,v,j_1,j_2)$, $S_o(t,v,j)$, $S_M(t,v,m,j)$, and $S_R(t,v,d,j)$ from the sets of possibilities $\{n_s^{(k_s)}(t,v,j_1,j_2)\}$, $\{S_o^{(k_o)}(t,v,j)\}$, $\{S_M^{(k_M)}(t,v,m,j)\}$, and $\{S_R^{(k_R)}(t,v,d,j)\}$. Determined by starting with situation at end of t-th time interval and simulating results for future time intervals under the assumption that $d_N^{(u)}(t') = D_N^{(u)}[t, j_u(t')]$ and $d_I^{(u)}(t') = D_I^{(u)}[t, j_u(t')]$ for $t' > t$ and all u. Here, for $t < t' \leq t + t_e(t)$, the $n_s(t',v,j_1,j_2)$, $S_o(t',v,j)$, $S_M(t',v,m,j)$, and $S_R(t',v,d,j)$ are determined from $e(t')$; in general, all possible selections for these strategies must be considered. For $t' > t + t_e(t)$, however, each of these strategies is uniquely specified on the basis of an efficiency criterion $e[t';t,t_e(t)]$ that is determined by the actual and anticipated results (all possible combinations of selections) for the situations existing at the end of time intervals up to and including $t + t_e(t)$

$g_C(v,d,m,j)$ = type of logistics item that is used for transportation in subsystem j of in-transit items of type v, damage level d, an needing maintenance of level m. Here in-transit items are grouped with respect to type, damage level, maintenance level needed, and subsystem location in such a manner that all items of a group can be handled by the same type of logistics item; hence many combinations of values for v,d,m,j may yield the same value for $g_C(v,d,m,j)$. Note that one or both of d and m are zero

$g_M(v,m,j)$ = type of logistics item used for maintenance of items of type v that are receiving maintenance at level m in subsystem j

$g_R(v,d,j)$ = type of logistics item used for repair of items of type v that are receiving repair of level d damage in subsystem j

$N_C(t,t',g_C,j)$ = anticipated number of in-transit items in subsystem j at the end of each of the time intervals $t + 1,\ldots,t + t' \leq T$ that are transported in this subsystem by logistics items of type g_C. This vector determined by starting with situation at end of t-th interval and simulating results for time intervals $t + 1,\ldots,t + t'$ under the assumption that $d_N^{(u)}(t'') = D_N^{(u)}[t,j_u(t'')]$ and $d_I^{(u)}(t'') = D_I^{(u)}[t,j_u(t'')]$ for $t + 1 \leq t'' \leq t + t'$ and all u

$N_M(t,t',g_M,j)$ = anticipated number of items in maintenance in subsystem j at the end of each of the time intervals $t + 1,\ldots,t + t' \leq T$ that are handled in maintenance in this subsystem by logistics items of type g_M. This vector determined by starting with situation at end of t-th interval and simulating results for time intervals $t + 1,\ldots,t + t'$ under the assumption that $d_N^{(u)}(t'') = D_N^{(u)}[t,j_u(t'')]$ and $d_I^{(u)}(t'') = D_I^{(u)}[t,j_u(t'')]$ for $t + 1 \leq t'' \leq t + t'$ and all u

$N_R(t,t',g_R,j)$ = anticipated number of items in repair in subsystem j at the end of each of the time intervals $t + 1,\ldots,t + t' \leq T$ that are handled in repair in this subsystem by items of type g_R. This vector determined by starting with situation at end of t-th interval and simulating results for time intervals $t + 1,\ldots,t + t'$ under the assumption that $d_N^{(u)}(t'') = D_N^{(u)}[t,j_u(t'')]$ and $d_I^{(u)}(t'') = D_I^{(u)}[t,j_u(t'')]$ for $t + 1 \leq t'' \leq t + t'$ and all u

$N(t,t',g,j)$ = anticipated number of logistics items of type g that are in operation (status 1) in subsystem j at the end of each of the time intervals $t + 1,\ldots,t + t' \leq T$. This vector determined by starting with situation at end of t-th interval and simulating results for time intervals $t + 1, \ldots, t + t'$ under the assumption that $d_N^{(u)}(t'') = D_N^{(u)}[t, j_u(t'')]$ and $d_I^{(u)}(t'') = D_I^{(u)}[t, j_u(t'')]$ for $t + 1 \le t'' \le t + t'$ and all u

$t_o[t, v, t^{(o)}, j^{(o)}, j_o(t)]$ = anticipated additional number of time intervals required, after t-th interval, for operational item of type v that began transit at the end of time interval $t^{(o)}$ from subsystem $j^{(o)}$ and which is now in subsystem $j_o(t)$ to complete transit in $j_o(t)$. Determined on basis of $N_C(t, t', g_C, j)$ and $N(t, t', g, j)$; value of $t_o[0, v, t^{(o)}, j^{(o)}, j_o(0)]$ is specified initial condition. Set equal to $T + 1 - t$ if transit in $j_o(t)$ not anticipated to be completed by end of T-th interval. Item leaves a transit subsystem and enters next subsystem specified by $S_o(t^{(o)}, v, j^{(o)})$ when $t_o[t, v, t^{(o)}, j^{(o)}, j_o(t)] = 0$

$j_o(t, v, t^{(o)}, j^{(o)})$ = subsystem location during t-th time interval for operational unit (status 2) of type v that began transit at the end of time interval $t^{(o)}$ from subsystem $j^{(o)}$

$t_M[t, v, m, t^{(M)}, j^{(M)}, j_M(t)]$ = anticipated additional number of time intervals required, after t-th interval, for item of type v that needs maintenance of level m, began transit at end of time interval $t^{(M)}$ from subsystem $j^{(M)}$, and is now in subsystem $j_M(t)$, to complete transit in $j_M(t)$. Determined on basis of $N_M(t, t', g_M, j)$ and $N(t, t', g, j)$; value of $t_M[0, v, m, t^{(M)}, j^{(M)},$

$j_M(0)]$ is specified initial condition. Set equal to $T + 1 - t$ if transit in $j_M(t)$ not anticipated to be completed by end of T-th interval. Item leaves a transit subsystem and enters next subsystem specified by $S_M(t^{(M)}, v, m, j^{(M)})$ when $t_M[t, v, m, t^{(M)}, j^{(M)}, j_M(t)] = 0$

$j_M(t, v, m, t^{(M)}, j^{(M)})$ = subsystem location during t-th time interval for item of type v that needs maintenance of level m and began transit at end of time interval $t^{(M)}$ from subsystem $j^{(M)}$

$t_R[t, v, d, t^{(R)}, j^{(R)}, j_R(t)]$ = anticipated additional number of time intervals required, after t-th interval, for item of type v that needs repair of damage level d, began transit for repair of this level of damage at the end of time interval $t^{(R)}$ from subsystem $j^{(R)}$, and is now in subsystem $j_M(t)$, to complete transit of $j_M(t)$. Determined on basis of $N_R(t, t', g_R, j)$ and $N(t, t', g, j)$; value of $t_R[0, v, d, t^{(R)}, j^{(R)}, j_R(0)]$ is specified initial condition. Set equal to $T + 1 - t$ if transit in $j_R(t)$ not anticipated to be completed by end of T-th interval. Item leaves a transit subsystem and enters next subsystem specified by $S_R(t^{(R)}, v, d, j^{(R)})$ when $t_R[t, v, d, t^{(R)}, j^{(R)}, j_R(t)] = 0$

$j_R(t, v, d, t^{(R)}, j^{(R)})$ = subsystem location during t-th time interval for item of type v that needs repair of level d damage and which began transit for repair of damage at this level at the end of time interval $t^{(R)}$ from subsystem $j^{(R)}$

$j_u^{(o)}(t)$ = location of u-th item during t-th time interval when this item operational and in transit (status 2) at end of (t - 1)-th interval

$t_u^{(o)}(t)$ = anticipated number of additional time intervals, after t-th, for completion of transit in subsystem $j_u^{(o)}(t)$ for u-th item, when this item in status 2 at end of (t - 1)-th interval. Determined on basis of $t_o[t,v,t^{(o)},j^{(o)},j_o(t)]$

$j_u^{(M)}(t)$ = location of u-th item during t-th time interval when this item in transit to maintenance (status 3) at end of (t - 1)-th interval

$t_u^{(M)}(t)$ = anticipated number of additional time intervals, after t-th, for completion of transit in subsystem $j_u^{(M)}(t)$ for u-th item, when this item in status 3 at end of (t - 1)-th interval. Determined on basis of $t_M[t,v,m,t^{(M)},j^{(M)},j_M(t)]$

$j_u^{(R)}(t)$ = location of u-th item during t-th time interval when this item in transit to repair (status 4 or 5) at end of (t - 1)-th interval

$t_u^{(R)}(t)$ = anticipated number of additional time intervals, after t-th, for completion of transit in subsystem $j_u^{(R)}(t)$ for u-th item, when this item in status 4 or 5 at end of (t - 1)-th interval. Determined on basis of $t_R[t,v,d,t^{(R)},j^{(R)},j_R(t)]$

$T_M(t,v,m,j)$ = anticipated number of time intervals to completion of maintenance for item of type v that begins maintenance of level m in subsystem j at the end of the t-th time interval. Determined on the basis of $N_M(t,t',g_M,j)$ and $N(t,t',g,j)$ for t' sufficiently large. Set equal to $T + 1 - t$ if maintenance not anticipated to be completed by end of T-th interval

$T_R(t,v,d,j)$ = anticipated number of time intervals to completion of repair for item of type v that begins repair of level d damage in subsystem j at the end of the t-th time interval. Determined on the basis of $N_R(t,t',g_R,j)$ and $N(t,t',g,j)$ for t' sufficiently large. Set equal to $T + 1 - t$ if repair not anticipated to be completed by end of T-th interval

$C_u^{(o)}(t)$ = function used for stating whether the u-th item is chosen to begin transit at the end of the t-th interval when it was in operation (status 1) at beginning of this time interval. Has value 0 if $s_u(t - 1) = 1$ but u-th item not chosen to begin transit; has value 1 if $s_u(t - 1) = 1$ and item chosen to begin transit; has value 2 otherwise

$\tau_u^{(o)}(t)$ = most recent time interval, up to and including t-th, such that $C_u^{(o)}[\tau_u^{(o)}(t)] = 1$

$\tau_u^{(M)}(t)$ = most recent time interval, up to and including t-th, such that $M_u\{T_o^{(u)}[\tau_u^{(M)}(t)]\} > 0$ but $M_u\{T_o^{(u)}[\tau_u^{(M)}(t) - 1]\} = 0$

$\tau_u^{(R)}(t)$ = most recent time interval, up to and including t-th, such that $d_u[\tau_u^{(R)}(t)] - d_u[\tau_u^{(R)}(t) - 1] > 0$

$\tau_m^{(u)}(t)$ = most recent time interval, up to and including t-th, such that $t_C^{(u)}[\tau_m^{(u)}(t) - 1] > 0$ or $M_u(T_o^{(u)}[\tau_m^{(u)}(t) - 1]) = 0$

$\tau_r^{(u)}(t)$ = most recent time interval, up to and including t-th, such that $t_C^{(u)}[\tau_r^{(u)}(t) - 1] > 0$ or $d_u[\tau_r^{(u)}(t) - 1] = 0$

$n_C(t,v,d,m,j)$ = number of items of type v, damage level d, and needing maintenance at level m that are in transit in subsystem j at end of t-th time interval

$n_M(t,v,m,j)$ = number of items of type v that are receiving maintenance of level m in subsystem j at end of t-th time interval

$n_R(t,v,d,j)$ = number of items of type v that are receiving repair of damage at level d in subsystem j at end of t-th time interval

$n_o(t,v,j)$ = number of items of type v in operation (status 1) in subsystem j at end of t-th time interval; $n_o(0,v,j)$ is specified initial condition

ASSUMPTIONS AND CONDITIONS

The model developed must satisfy some assumptions if the simulation is to furnish an acceptable approximation to the operation of the system being considered. These assumptions, which are concerned with the approximation of a continuous situation by a discrete situation, can be stated as follows:

(a) The basic time interval chosen for use is small enough to yield a reasonable approximation to the situation of continuous time.

(b) The basic time interval and the subsystems are such that the random damage received in one subsystem can be considered independent of the random damage received in any other subsystem during the same time interval.

(c) The subsystems are uncomplicated enough for their required properties to be satisfied to a reasonable approximation. That is, the use of these subsystems, in conjunction with the conditions and relationships specified by the model, furnishes an acceptable representation of the system behavior when the other assumptions hold.

Whether assumptions (a)-(c) are acceptable for a given-size basic time interval and a specified set of subsystems depends on the system being approximated and on the conditions and relationships imposed by the model.

The functional relationships used in specifying the mathematical model are stated in the next section. Separately, some conditions are adopted for the operation of the model. These conditions serve two purposes. First, some fundamental logistics priorities are specified. Second, the simulation model is simplified without much curtailment of its generality and applicability. The conditions imposed are

1. After completion of maintenance or repair, an item is considered to be operational (status 1) for at least one time interval.
2. Repair of damage takes precedence over maintenance. When an item needing maintenance or receiving maintenance is damaged, the item first receives repair. The required maintenance is then performed as if it were first coming due in the subsystem where the repair was completed. Functionally, this is accomplished by slightly reducing the value of $T_{\odot}^{(u)}(t)$, which has a value which specifies that maintenance is due, when the item becomes damaged; then maintenance will immediately become due as the item becomes operational at the end of repair. Thus, the model is such that an item can not simultaneously be damaged and need maintenance.
3. After a change of status because of the need for maintenance or the need for repair, an item is considered to be in transit (perhaps in the same subsystem) for at least one time interval before maintenance or repair is started.
4. For each combination of values for time interval, subsystem location, type of item, level of damage, and maintenance level needed, the scheduled transfer is to a single destination subsystem and by use of a single route (specified by the in-transit subsystems and their order in the transit).

5. If an item becomes due for maintenance or receives damage while in transit, it is treated as if it were an item (not in transit) of the subsystem where this status change occurred.
6. If a damaged item becomes further damaged, it is treated as if the totality of damage were just received in the subsystem where it is now located. Any repair already received for the previous damage level is not considered. However, the repair received is taken into consideration in deciding how much the values of $d_N^{(u)}(t)$ and $d_I^{(u)}(t)$ are to increase the value of $d_u(t - 1)$.
7. Changes in the characteristics of an item that occur during a given time interval do not have any effect on the other items until at least one time interval later.
8. An item stays in each of its subsystem locations for at least one time interval.

These conditions are used, sometimes only implicitly, in the statement of the functional relationships among the quantities occurring in the mathematical model.

FUNCTIONAL RELATIONS AND MATHEMATICAL MODEL

The mathematical model for the simulation consists of the statement of a method whereby knowledge of all the pertinent quantities up to and including the end of the $(t - 1)$-th time interval can be used to evaluate these quantities at the end of the t-th interval. This model, combined with the given initial conditions for the system, can be used for simulating the system operation over all the time intervals considered. Here the pertinent quantities for each time interval are those defined in the DEFINITION OF NOTATION.

As already mentioned, the general simulation model presented is not of a detailed nature, with the other values that are used to determine a given quantity only being identified in many cases. That is, in the statement of the model, the evaluation of a given quantity is often expressed in the form of an unspecified function of specified quantities at stated times. Thus the other values which are sufficient for evaluation of this quantity are stated but the form of the function used in the evaluation is not always stated. Of course, for a simulation of any given system, these functions would be specified on the basis of the properties of the system considered.

This section fulfills a dual purpose. First, for each quantity considered, the other values that are sufficient for determination of this quantity are specified. Second, the order in which functional relations for quantities are presented furnishes a statement of the mathematical model for the simulation. That is, this order is such that the specified values used for determining a quantity are always previously evaluated, on the basis of quantities which have already occurred in the ordering and/or quantities for preceding time intervals. In situations involving anticipated future behavior of the system, the simulation is projected into the future but is based exclusively on values that have already been determined.

The quantities $n_o(t,v,j)$, $n_C(t,v,d,m,j)$, $n_M(t,v,m,j)$, and $n_R(t,v,d,j)$ are determined by direct examination of the situation at the end of the t-th time interval, while $C_u^{(o)}(t)$ is determined by this examination and the value of $s_u(t-1)$. The v_u, $m_u^{(s)}$, $g_C(v,d,m,j)$, $g_M(v,m,j)$, $g_R(v,d,j)$, and $M_u[T']$ do not depend on the time interval considered while the $B_v(t,j)$ are specified independently of the simulation. The values of T, U, V, L, J, M, and D are specified. Some of the other quantities were defined in a functional form in the DEFINITION OF NOTATION. The remaining quantities that occur in the simulation model depend on the following forms of functional relationships:

$$T_M(t,v,m,j) = T_M\{t,v,m,j;T_M(t-1,v,m,j),N_M[t-1,T_M(t-1,v,m,j),$$
$$g_M(v,m,j),j],N[t-1,T_M(t-1,v,m,j),g_M(v,m,j),j]\}$$

$$T_R(t,v,d,j) = T_R\{t,v,d,j;T_R(t-1,v,d,j),N_R[t-1,T_R(t-1,v,d,j),$$
$$g_R(v,d,j),j],N[t-1,T_R(t-1,v,d,j),g_R(v,d,j),j]\}$$

$$j_o(t,v,t^{(o)},j^{(o)}) = j_o\{t,j_o(t-1,v,t^{(o)},j^{(o)}),t_o[t-1,v,t^{(o)},j^{(o)},$$
$$j_o(t-1,v,t^{(o)},j^{(o)})],S_o(t^{(o)},v,j^{(o)})\} \text{ for } t$$
$$t > t^{(o)} \text{ and equals } j^{(o)} \text{ for } t = t^{(o)}$$

$$t_o[t,v,t^{(o)},j^{(o)},j_o(t,v,t^{(o)},j^{(o)})] = t_o\{t,v,t^{(o)},j^{(o)},j_o(t,v,t^{(o)},j^{(o)});$$
$$t_o[t-1,v,t^{(o)},j^{(o)},j_o(t-1,v,t^{(o)},j^{(o)})],$$
$$N_C(t-1,t_o[t-1,v,t^{(o)},j^{(o)},j_o(t-1,v,t^{(o)},$$
$$j^{(o)})],g_C(v,0,0,j^{(o)}),j^{(o)}),N(t-1,$$
$$t_o[t-1,v,t^{(o)},j^{(o)},j_o(t-1,v,t^{(o)},j^{(o)})],$$
$$g_C(v,0,0,j^{(o)}),j^{(o)})\}$$

$$j_M(t,v,m,t^{(M)},j^{(M)}) = j_M\{t,j_M(t-1,v,m,t^{(M)},j^{(M)}),t_M[t-1,$$
$$v,m,t^{(M)},j^{(M)},j_M(t-1,v,m\ t^{(M)},j^{(M)})],$$
$$S_M(t^{(M)},v,m,j^{(M)})\} \text{ for } t > t^{(M)} \text{ and}$$
$$\text{equals } j^{(M)} \text{ for } t = t^{(M)}$$

$$t_M[t,v,m,t^{(M)},j^{(M)},j_M(t,v,m,t^{(M)},j^{(M)})] = t_M\{t,v,m,t^{(M)},j^{(M)},j_M(t,v,m,t^{(M)},j^{(M)});$$
$$t_M[t-1,v,m,t^{(M)},j^{(M)},j_M(t-1,v,m,t^{(M)},j^{(M)})],$$
$$N_C(t-1,t_M[t-1,v,m,t^{(M)},j^{(M)},j_M(t-1,v,m,$$
$$t^{(M)},j^{(M)})],g_C(v,0,m,j^{(M)}),j^{(M)}),N(t-1,$$
$$t_M[t-1,v,m,t^{(M)},j^{(M)},j_M(t-1,v,m,t^{(M)},j^{(M)})],$$
$$g_C(v,0,m,j^{(M)}),j^{(M)})\}$$

$$j_R(t,v,d,t^{(R)},j^{(R)}) = j_R\{t,j_R(t-1,v,d,t^{(R)},j^{(R)}),t_R[t-1,v,d,$$
$$t^{(R)},j^{(R)},j_R(t-1,v,d,t^{(R)},j^{(R)})],S_R(t^{(R)},$$

$v, d, j^{(\bar{R})})\}$ for $t > t^{(\bar{R})}$ and equals

$j^{(\bar{R})}$ for $t = t^{(\bar{R})}$

$$t_R[t, v, d, t^{(R)}, j^{(\bar{R})}, j_R(t, v, d, t^{(R)}, j^{(\bar{R})})] = t_R\{t, v, d, t^{(R)}, j^{(\bar{R})}, j_R(t, v, d, t^{(R)}, j^{(\bar{R})}); t_R[t-1, v, d, t^{(R)}, j^{(\bar{R})}, j_R(t-1, v, d, t^{(R)}, j^{(\bar{R})})], N_C(t-1, t_R[t-1, v, d, t^{(R)}, j^{(\bar{R})}, j_R(t-1, v, d, t^{(R)}, j^{(\bar{R})})], g_C(v, d, 0, j^{(\bar{R})}), j^{(\bar{R})}), N(t-1, t_R[t-1, v, d, t^{(R)}, j^{(\bar{R})}, j_R(t-1, v, d, t^{(R)}, j^{(\bar{R})})], g_C(v, d, 0, j^{(\bar{R})}), j^{(\bar{R})})\}$$

$$j_u^{(o)}(t) = j_o\{t, v_u, \tau_u^{(o)}(t), j_u[\tau_u^{(o)}(t)]\}$$

$$t_u^{(o)}(t) = t_o\{t, v_u, \tau_u^{(o)}(t), j_u[\tau_u^{(o)}(t)], j_u^{(o)}(t)\}$$

$$j_u^{(M)}(t) = j_M[t, v_u, M_u\{T_o^{(u)}[\tau_u^{(M)}(t)]\}, \tau_u^{(M)}(t), j_u\{\tau_u^{(M)}(t)\}]$$

$$t_u^{(M)}(t) = t_M[t, v_u, M_u\{T_o^{(u)}[\tau_u^{(M)}(t)]\}, \tau_u^{(M)}(t), j_u\{\tau_u^{(M)}(t)\}, j_u^{(M)}(t)]$$

$$j_u^{(R)}(t) = j_R\{t, v_u, d_u[\tau_u^{(R)}(t)], \tau_u^{(R)}(t), j_u[\tau_u^{(R)}(t)]\}$$

$$t_u^{(R)}(t) = t_R\{t, v_u, d_u[\tau_u^{(R)}(t)], \tau_u^{(R)}(t), j_u[\tau_u^{(R)}(t)], j_u^{(R)}(t)\}$$

$$j_u(t) = j_u(t-1) \text{ when } s_u(t-1) = 1,6, \text{ or } 7$$

$$= j_u^{(o)}(t) \text{ when } s_u(t-1) = 2$$

$$= j_u^{(M)}(t) \text{ when } s_u(t-1) = 3$$

$$= j_u^{(R)}(t) \text{ when } s_u(t-1) = 4 \text{ or } 5$$

$$P_N^{(u)}(d;t) = P_N\{d; t, v_u, j_u(t), m_u^{(s)}, n_o[t-1, v_u, j_u(t)], n_C[t-1, v_u, d_u(t-1), M_u\{T_o^{(u)}(t-1)\}, j_u(t)], n_M[t-1, v_u, M_u\{T_o^{(u)}(t-1)\} j_u(t)], n_R[t-1, v_u, d_u(t-1), j_u(t)]; d_u(t'), s_u(t'), j_u(t'), T_o^{(u)}(t'); t' = 0,1,\ldots,t-1\}$$

$$P_I^{(u)}(d;t) = P_I[d; t, v_u, j_u(t); d_u(t'), s_u(t'), j_u(t'), n_o(t', v, j), n_C(t', v, d', m, j), n_M(t', v, m, j), n_R(t', v, d', j); (t' = 0,\ldots,t-1; j = 1,\ldots, J; v = 1,\ldots,V; d' = 0,1,\ldots,D; m = 0,1,\ldots,M)]$$

$$d_u(t) = d\{u, d_u(t-1), s_u(t-1), d_N^{(u)}(t), d_I^{(u)}(t), t_r^{(u)}(t-1), T_R[t, v_u, d_u(t-1), j_u(t)]\}$$

$$T_o^{(u)}(t) = T_o[t, v_u, T_o^{(u)}(t-1), s_u(t-1), d_u(t), d_u(t-1), j_u(t)]$$

$$M_u[T'] = M(u,T',m_u^{(s)})$$

$$\begin{aligned}
t_C^{(u)}(t) = t_C[&t,v_u,t_C^{(u)}(t-1),s_u(t-1),d_u(t),d_u(t-1),M_u\{T_o^{(u)}(t)\},\\
&M_u\{T_o^{(u)}(t-1)\},j_u(t),c_u^{(o)}(t),\tau_u^{(o)}(t),\tau_u^{(M)}(t),\tau_u^{(R)}(t),\\
&t_u^{(o)}(t),t_u^{(M)}(t),t_u^{(R)}(t),S_o\{\tau_u^{(o)}(t),v_u,j_u[\tau_u^{(o)}(t)]\},\\
&S_M\{\tau_u^{(M)}(t),v_u,M_u[T_o^{(u)}(t)],j_u[\tau_u^{(M)}(t)]\},S_R\{\tau_u^{(R)}(t),v_u,\\
&d_u[\tau_u^{(R)}(t)],j_u[\tau_u^{(R)}(t)]\},N_C\{t-1,t_C^{(u)}(t-1),g_C[v_u,d_u(t),\\
&M_u\{T_o^{(u)}(t)\},j_u(t)],j_u(t)\},N\{t-1,t_C^{(u)}(t-1),g_C[v_u,d_u(t),\\
&M_u\{T_o^{(u)}(t)\},j_u(t)],j_u(t)\}]
\end{aligned}$$

$$\begin{aligned}
t_m^{(u)}(t) &= 0 \text{ if } t_C^{(u)}(t) > 0 \text{ or if } M_u[T_o^{(u)}(t) = 0\\
&= t_m[t,v_u,t_m^{(u)}(t-1),s_u(t-1),M_u\{T_o^{(u)}(t)\},j_u\{\ _m^{(u)}(t)\},\\
&\qquad N_M\{t-1,t_m^{(u)}(t-1),g_M(v_u,M[T_o^{(u)}(t)],j_u[\tau_m^{(u)}(t)]),\\
&\qquad j_u[\tau_m^{(u)}(t)]\},N\{t-1,t_m^{(u)}(t-1),g_M(v_u,M[T_o^{(u)}(t)],\\
&\qquad j_u[\tau_m^{(u)}(t)]),j_u[\tau_m^{(u)}(t)]\}] \text{ otherwise}
\end{aligned}$$

$$\begin{aligned}
t_r^{(u)}(t) &= 0 \text{ if } t_C^{(u)}(t) > 0 \text{ or if } d_u(t) = 0\\
&= t_r[t,v_u,t_r^{(u)}(t-1),s_u(t-1,d_u(t),j_u\{\tau_r^{(u)}(t)\},N_R\{t-1,t_r^{(u)}(t-1),\\
&\qquad g_R(v_u,d_u(t),j_u[\tau_r^{(u)}(t)]),j_u[\tau_r^{(u)}(t)]\},N\{t-1,t_r^{(u)}(t-1),\\
&\qquad g_R(v_u,d_u(t),j_u[\tau_r^{(u)}(t)]),j_u[\tau_r^{(u)}(t)]\}] \text{ otherwise}
\end{aligned}$$

$$t_o^{(u)}(t) = 0 \text{ when } d_u(t) = 0 \text{ and } M_u[T_o^{(u)}(t)] = 0$$

$$= t_C^{(u)}(t) + T_M[t+t_C^{(u)}(t), v_u, M_u[T_o^{(u)}(t)], j_u^{(M)}[t+t_C^{(u)}(t)]]$$
$$\text{when } M_u[T_o^{(u)}(t)] > 0 \text{ and } t_C^{(u)}(t) > 0$$

$$= t_C^{(u)}(t) + T_R[t+t_C^{(u)}(t), v_u, d_u(t), j_u^{(R)}[t+t_C^{(u)}(t)]] \text{ when}$$
$$d_u(t) > 0 \text{ and } t_C^{(u)}(t) > 0$$

$$= t_m^{(u)}(t) \text{ when } M_u[T_o^{(u)}(t)] > 0 \text{ and } t_C^{(u)}(t) = 0$$

$$= t_r^{(u)}(t) \text{ when } d_u(t) > 0 \text{ and } t_C^{(u)}(t) = 0$$

$$j_m^{(u)}(t) = 0 \text{ when } M_u[T_o^{(u)}(t)] = 0 \text{ or } t_C^{(u)}(t) > 0$$

$$= j_u[\tau_m^{(u)}(t)] \text{ otherwise}$$

$$j_r^{(u)}(t) = 0 \text{ when } d_u(t) = 0 \text{ or } t_C^{(u)}(t) > 0$$

$$= j_u[\tau_r^{(u)}(t)] \text{ otherwise}$$

$$s_u(t) = 1 \text{ when } t_o^{(u)}(t) = 0, t_C^{(u)}(t) = 0$$

$$= 2 \text{ when } t_o^{(u)}(t) = 0, t_C^{(u)}(t) > 0$$

$$= 3 \text{ when } t_o^{(u)}(t) > 0, t_C^{(u)}(t) > 0, M_u[T_o^{(u)}(t)] > 0$$

$= 4$ when $t_o^{(u)}(t) > 0, t_C^{(u)}(t) > 0, d_u(t) > d_u(t-1)$

$= 5$ when $t_o^{(u)}(t) > 0, t_C^{(u)}(t) > 0, d_u(t) > 0, d_u(t)$
$= d_u(t-1)$

$= 6$ when $t_o^{(u)}(t) > 0, t_C^{(u)}(t) = 0, M_u[T_o^{(u)}(t)] > 0$

$= 7$ when $t_o^{(u)}(t) > 0, t_C^{(u)}(t) = 0, d_u(t) > 0$

$q_u[t, v_u, j_u(t), j_2] = q\{t, u, v_u, j_u(t), j_2, s_u(t-1), d_u(t), T_o^{(u)}(t); T_o^{(u')}(t),$
$m_s^{(u')}$, for all u', such that $v_{u'} = v_u, j_{u'}(t) = j_u(t),$
$s_{u'}(t-1) = 1, d_{u'}(t) = 0$ and $M_{u'}[T_o^{(u')}(t)] = 0\}$

$n_1(t,v,j) = n_1\{t, v, j, n_o(t-1, v, j); v_u, d_u(t), M_u[T_u^{(o)}(t)]$, for all
u, such that $v_u = v, j_u(t) = j$, and $s_u(t-1) = 1\}$

$n_a(t,v,j_1,j_2) = \min\{n_s(t,v,j_1,j_2), \max[0, n_1(t,v,j_1) - B_v(t,j_1)]\}$

$e(t)$ = function that can depend on any of the simulated actual results (up to end of t-th interval) and any of the anticipated future results for all possible combined choices for the present and future strategies

$e[t'; t, t_e(t)]$ = function that can depend on any of the actual results (up to end of t-th interval) and any of the anticipated future results that are obtained on basis of situation at end of t'-th interval, where $t' \leq t + t_e(t)$

$n_s(t,v,j_1,j_2), S_o(t,v,j), S_M(t,v,m,j), S_R(t,v,d,j)$

= strategies that are chosen from all combinations of possibilities by anticipating future behavior of system, along lines specified in the DEFINITION OF NOTATION, and use of $e(t)$; determines $J_o(t,v,j)$

$N_C(t,t',g_C,j), N_M(t,t',g_M,j), N_R(t,t',g_R,j), N(t,t',g,j)$

= functions that are directly obtained by anticipating future behavior of system (as outlined the DEFINITION OF NOTATION) using the selected strategies

This completes the statement of the functional relationships among the various quantities that occur in the simulation model presented. The mathematical model furnished by these relationships, and their order of presentation, seems to be sufficiently general for use in the investigation of many types of logistics situations.

UNCLASSIFIED

System Development Corporation,
Santa Monica, California
A GENERAL SIMULATION MODEL FOR
LOGISTICS OPERATION IN A RANDOMLY-
DAMAGED SYSTEM.
Scientific rept., SP-116, by J. E. Walsh.
10 January 1960, 39p.
Unclassified report

DESCRIPTORS: Logistics. Simulation.
Damage. Monte Carlo Method.

Describes a simulation model for logistics operation in a system in which natural attrition and induced attrition occur. Reports that the capability of the

UNCLASSIFIED

UNCLASSIFIED

logistics operation is examined by simulating the operation of the entire system for a specified time period. Further reports that to obtain the simulation model, the overall system is divided into subsystems and that attrition is introduced by a Monte Carlo procedure. Repetitions of this simulation furnish an indication of the effects of statistical variation. Concludes that the mathematical model furnished by the relationships of the various quantities that occur in the simulation model presented, and their order of presentation, seems to be sufficiently general for use in the investigation of many types of logistics situations.

UNCLASSIFIED